Integral Substitution

Substitution w/ Indefinite Integrals: Let $u = g(x)$, where $g'(x)$ is cont over an interval, let $f(x)$ be cont over the corresponding range of g, & let $F(x)$ be an antiderivative of $f(x)$

$$\Rightarrow \int f[g(x)]\, g'(x)\, dx = \int f(u)\, du = F(u) + C = F(g(x)) + C$$

Ex. Use substitution to find $\int 6x(3x^2+4)^4 dx$

$u = 3x^2 + 4$

$u' = 6x\, dx$

$$\int 6x(3x^2+4)^4 dx$$

$$\int u^4 du = \frac{u^5}{5} + C = \boxed{\frac{(3x^4+4)^5}{5} + C}$$

Ex. Use substitution to evaluate $\int \frac{\sin t}{\cos^3 t} dt$

$u = \cos t$

$u' = -\sin t\, dt$

$$\int \frac{\sin t}{\cos^3 t} dt$$

$$-\int \frac{1}{u^3} du = -\int u^{-3} du = -\frac{1}{-2u^2} + C = \boxed{\frac{1}{2\cos^2 t} + C}$$

Ex. Use substitution to solve $\int \frac{x}{\sqrt{x-1}} dx$

now we can continue since we can now substitute for x

$u = x - 1 \longrightarrow x = u + 1$

$du = dx$

still need a way to get rid of x in numerator

$$\int \frac{x}{\sqrt{x-1}} dx$$

$$\int \frac{u+1}{\sqrt{u}} du = \int \frac{u}{u^{\frac{1}{2}}} + \frac{1}{u^{\frac{1}{2}}} du = \int u^{\frac{1}{2}} + u^{-\frac{1}{2}} du$$

$$= \frac{2\sqrt{u^3}}{3} + 2\sqrt{u} + C = \boxed{\frac{2\sqrt{(x-1)^3}}{3} + 2\sqrt{x-1} + C}$$

Substitution w/ Definite Integrals: Let $u = g(x)$ & let g' be cont over range of $u = g(x) \Rightarrow \int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$

Ex. Use substitution to evaluate $\int_0^1 x^2(1+2x^3)^5 dx$

$u = 1 + 2x^3$

$du = 6x^2 dx$

x	0	1
u	1	3

use eqn

$$\frac{1}{6}\int_1^3 u^5 du = \frac{1}{6}\left(\frac{u^6}{6}\right)\Bigg|_1^3$$

$$= \frac{1}{6}\left(\frac{(3)^6}{6} - \frac{(1)^6}{6}\right) = \boxed{\frac{182}{9}}$$

More Practice

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Ex. Use substitution to evaluate $\int_0^1 x e^{4x^2+3}\,dx$

$$\frac{1}{8}\int_3^7 e^u\,du = \frac{1}{8}\left(e^u\right)\Big|_3^7 = \boxed{\frac{e^7 - e^3}{8}}$$

$u = 4x^2+3$

$u' = 8x\,dx$

x	0	1
u	3	7

Works Cited

Strang, Gilbert, et al. "1.3 the Fundamental Theorem of Calculus - Calculus Volume 2 | OpenStax." *Openstax.org*, 30 Mar. 2016, openstax.org/books/calculus-volume-2/pages/1-3-the-fundamental-theorem-of-calculus. Accessed 2 Aug. 2022.